Fall 2009 Math 245 Exam 3

Please read the following directions:

Please write legibly, with plenty of white space. Please put your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use your book and/or notes, but no calculators or other aids. This exam will last 50 minutes; pace yourself accordingly. If you are done early, you may leave – but NOT during the last five minutes of the exam, during which you are asked to remain quiet and in your seat. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Total:	50		100

Problem 1. Carefully define the following terms:

    a. codomain

    b. injection

    c. partially ordered set

    d. l.u.b.

    e. strong pigeonhole principle

Problem 2. Prove or disprove the following statement. For all functions $f : \mathbb{N} \to \mathbb{N}$, if f is injective then f is surjective.

Problem 3. Find all equivalence relations on $A = \{x, y\}$.

Problem 4. Find all posets on $A = \{x, y\}$.

Problem 5. Find all functions $f : A \to A$, for $A = \{x, y\}$.

Problem 6. How many relations are there on $A = \{x, y\}$? Give two examples, neither of which are equivalence relations, posets, or functions.

Problem 7. Solve the recurrence given by $a_0 = 3, a_1 = -3, a_n = -4a_{n-1} - 4a_{n-2} (n \geq 2)$.

Problem 8. Prove that $f : \mathbb{N} \to \mathbb{Z}$ is injective, where $f(n) = \begin{cases} n/2 & n \text{ even} \\ (1-n)/2 & n \text{ odd} \end{cases}$.

Problem 9. We define a lattice on $A = \mathbb{R} \times \mathbb{R}$ as follows. For $x = (x_1, x_2), y = (y_1, y_2)$, elements of A, we say $x \leq y$ if $(x_1 \leq y_1 \text{ AND } x_2 \leq y_2)$. For $x = (0.5, 3), y = (4, 1)$, find l.u.b.(x, y) and g.l.b.(x, y).

Problem 10. Find a finite-state automaton on $\Sigma = \{a, b\}$ that recognizes those words with exactly one b (and no other words).